Topology Alteration for Virtual Sculpting using Spatial Deformation

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ABSTRACT

Virtual sculpting enables the creation of computer models by emulating traditional sculpting. It can be implemented using spatial deformation, an interactive versatile modelling technique. Unfortunately, spatial deformation is limited to topology preserving warping. This is overcome by space-time objects, a variant of spatial deformation, which alters topology by *extruding* an object into 4-D, *deforming* the 4-D object and *extracting* a topologically altered object. However, they are specifically targeted to animation. In this paper, we adapt space-time objects to interactive modelling by: employing a tetrahedral rather than parallelepiped representation; exploiting coherence during the constant projection into four dimensions; and limiting projection to the portions of an object undergoing topology changes and thereby producing simpler triangulations of undeformed regions. Each of these adaptations is discussed in the context of the space-time object stages: extrusion, deformation and extraction. We also present preliminary results demonstrating the efficiency of our improvements.

Categories and Subject Descriptors

I.3.5 [**Computational Geometry and Object Modelling**]: Modeling Packages—*Virtual Sculpting, Topology Alteration*

General Terms

Algorithms, Experimentation

Keywords

Topology Alteration, Virtual Sculpting, Computer Modelling, Computer Graphics

1. INTRODUCTION

Virtual sculpting is a computational modelling technique that emulates traditional sculpting [5]. It offers several advantages over the costly and cumbersome laser scanning of clay models [12]: an artist can edit models without needing to resculpt and controls the entire creation cycle not just the physical sculpting. Errors in laser scanning, such as topological noise [9], are eliminated.

Spatial deformation [13], an *interactive* aesthetically-pleasing versatile modelling technique, can be used to implement virtual sculpting. Unfortunately, conventional spatial deformation cannot make topological changes to an object, which include operations such as hole creation and surface separation. In this sense it is more limited than laser scanning. It is therefore vital to develop a topology alteration scheme if virtual sculpting is to successfully compete with traditional sculpting. Such a scheme should maintain the desirable qualities of spatial deformation that make it suitable for virtual sculpting.

This problem is solved, in the context of animation, by space-time objects (summarised in section 2). However, they are ill-suited to the demands of virtual sculpting. Our contribution is to adapt space-time objects to interactive modelling by: employing a tetrahedral rather than parallelepiped representation (section 3); exploiting coherence during the constant projection into four dimensions (section 4); and limiting projection to those portions of an object undergoing topology changes (section 5). We also present preliminary results (section 6) indicating the efficiency of our improvements and identify some crucial issues for further research (section 7).

2. BACKGROUND

Successful virtual sculpting reduces the difficulty of 3-D computerised modelling by drawing on a user's familiarity with traditional sculpting. This is not a slavish simulation since it is augmented by the advantages of a computer-based implementation (e.g., undo operations, multiple viewing angles). The following properties should be supported [6]:

- **Interactivity** - In a physical medium, such as clay, changes occur instantaneously. Virtual sculpting should therefore provide interactive feedback[1].
- **Flexibility** - It should be possible to replicate the full range of shapes possible under conventional sculpting.

[1]Generally accepted to be between ten and fifteen updates-per-second.

- **Aesthetics** - The final object should appear smooth, exhibiting C_1 continuity[2]. Physical properties such as volume preservation and gravity are ignored.

- **Correctness** - The model must be geometrically correct. For instance, it should not self-intersect, since this is physically unrealistic, aesthetically annoying and may cause some renderers to fail.

The flexibility attribute requires that hole creation, splitting and merging of objects be possible. This emulates the friability of clay which can be pierced with holes, separated into pieces and joined into a single whole. These operations are collectively called topology alteration and require a *connectivity change* of the object. Their implementation requires a balance with other sculpting properties. Specifically, topology alteration must leave the object in an aesthetically pleasing and correct state, as well as being interactive.

We have chosen to implement virtual sculpting using Directly Manipulated Free-Form Deformation (FFD) [14], a form of spatial deformation that is flexible, intuitive and provides interactive feedback. Spatial deformation warps an object by altering the positions of sample points on its surface. It is a two stage process: sample points are embedded in the ambient space, which is then warped, transmitting the consequent deformation to the embedded points. Much research has been done on this topic, and the interested reader is referred to overviews by Bechmann [3] and Milliron *et al.* [13]. Spatial deformation only affects geometry, leaving *connectivity unchanged*. It is thus inherently topology preserving and requires substantial extension to support topology alteration.

Aubert and Bechmann [2] combine space-time objects and spatial deformation to achieve topology alteration for computer animation. Their method extrudes an n-dimensional object into an $(n{+}1)$-dimensional space-time object. This supports aesthetic topology changes, unlike Constructive Solid Geometry [10] which tends to introduce sharp non-C_1 edges. There are three specific stages in using space-time objects to alter topology (refer to Figure 1):

1. **Extrusion** - The object is projected from n- to $(n{+}1)$-dimensions by stacking and connecting multiple versions of the object, offset as layers in the higher dimensional space. For example, a circle in 2-D becomes a 3-D space-time cylinder by joining together a series of translated circles. The original object is embedded as a layer within the space-time object. The number of layers and their separation defines the 'height' of the space-time object.

2. **Deformation** - The space-time object is warped, using spatial deformation. The topology of the space-time object is unchanged but this does not necessarily hold for the original embedded object.

3. **Extraction** - An altered n-dimensional object is extracted from the deformed space-time object by intersecting it with a hyper-plane.

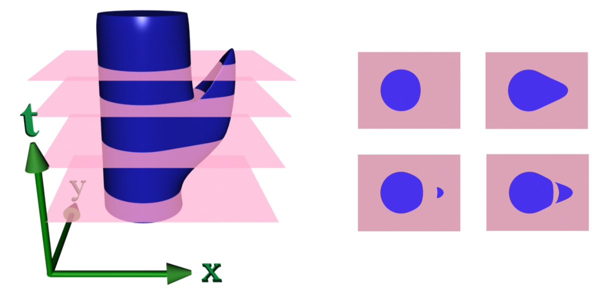

Figure 1: Component separation of a 2-D circle. The circle is extruded into a 'cylindrical' space-time object. The space-time object is then deformed [left] and the extracted object exhibits gradual surface separation [right].

Aubert and Bechman's technique is targeted to animation, where interactivity is less crucial than modelling. In this paper we present, three improvements aimed at Virtual Sculpting:

- **Tetrahedral Representation** - Aubert and Bechmann employ a 3-D polygon-mesh with quadrilateral faces and, consequently, their 4-D space-time object has parallelepiped faces. This is problematic because the extracted quadrilateral faces are not guaranteed to be planar or convex. They must be broken down into triangles for rendering and this cannot be done in an unambiguous manner [2]. We consistently employ 3-D triangles and 4-D tetrahedra which overcomes any ambiguity.

- **Constant Extrusion** - It suffices in modelling applications for extrusion to be constant in the n+1-th dimension. We exploit the many forms of resulting coherence to improve efficiency.

- **Localised Topology Alteration** - For modelling it is only necessary to extrude, deform and extract that localised portion of an object undergoing topology alteration. Unlike Aubert and Bechmann, who transform the entire object, we concentrate only on the necessary subset.

3. TETRAHEDRAL REPRESENTATION

During extrusion, vertices are connected to their counterparts in consecutive layers. 3-D objects represented by a surface of connected triangles are extruded into 4-D objects made up of a surface of 3-D triangular prisms, which are then subdivided into tetrahedra (see Figure 2). Using tetrahedra provides two distinct advantages [3] [1]:

- **Convexity** - A tetrahedron is by definition the simplest convex set in three dimensions and never becomes non-convex under spatial deformation.

[2]Existence and continuity of first derivatives.
[3]We assume that a tetrahedron is non-degenerate, i.e., its vertices have unique spatial positions.

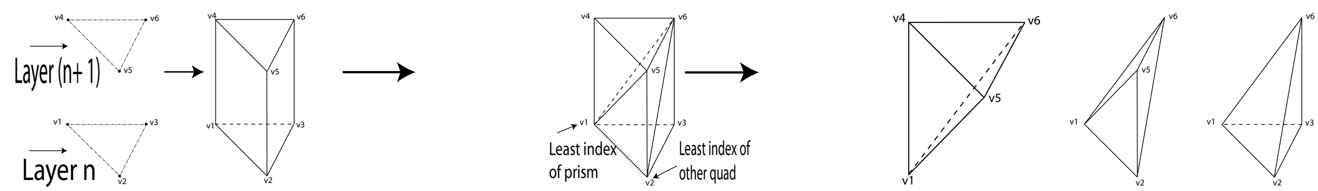

Figure 2: [Left] Vertices belonging to a triangle face are connected together in consecutive layers during extrusion to form a prism, [Middle] the least vertex index determines splitting of the prism quads into triangles, [Right] the resulting tetrahedral subdivision.

- **Planarity** - The vertices of a tetrahedron in 4-D define linearly indepedenent vectors which are the basis for a hyperplane, in the same way as a triangle defines a plane in 3-D. They are therefore implicitly planar.

The tetrahedral subdivision of a 4-D prism surface, should have the following properties:

- **Two-Manifold [10]** - Both the 4-D extruded surface and the 3-D extracted surface are two-manifold, implying that any point neighbourhood on the surface is topologically equivalent to a disk. This avoids problems such as dangling edges and T-vertices.
- **Minimal** - In the interests of memory conservation and computation efficiency, subdivision should result in the least number of tetrahedra. In particular, adding an extra (Steiner) vertex to accomplish subdivision must be avoided.

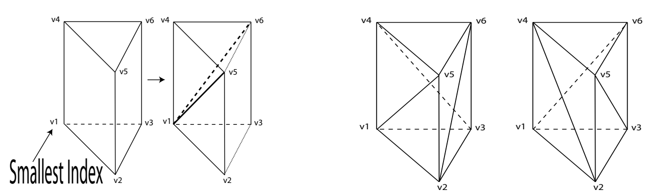

Figure 3: [Left] Splitting along a least vertex index ensures that two diagonals share a vertex, [Right] the two Schoñhardt polyhedra resulting from splitting quad faces with diagonals that do not share a vertex.

Tetrahedral subdivision involves splitting the quadrilateral faces of prisms into triangles. There are three quads per prism, each with two possible diagonal splits, giving rise to eight configurations. Of these eight configurations, six can be subdivided into three tetrahedra [11]. The remaining two prisms, known as Schoñhardt polyhedra, lead to a subdivision with four tetrahedra, and require a Steiner vertex [4]. This occurs when the diagonals used for splitting quads do not meet at a common vertex [4, 11] (see Figure 3). To ensure minimalism, we avoid Schoñhardt polyhedra.

In the interests of consistency and a two-manifold surface, a means of uniquely identifying and ordering vertices is needed. It is natural to choose the vertex indices of the polygon-mesh structure for this purpose. Quads are split by identifying the vertex with the smallest index and then connecting it to the corresponding diagonal vertex (refer to Figure 2). This is accomplished by identifying the least index in the prism and using it as a source for the diagonal split of the two incident quads. This eliminates Schoñhardt polyhedra. It only remains to find the least index for the third quad and split accordingly. This splitting procedure ensures that neighbouring prisms are tetrahedronalised in a consistent two-manifold manner.

After consistent quad splitting, it becomes trivial to subdivide prisms into tetrahedra. There are only six cases to consider, depending on the diagonals of the quads (see Figure 2 for an example). For efficiency, a look-up table is used to determine a prism's tetrahedral subdivision.

The overhead of the 4-D deformation phase is not increased because Steiner vertices are not introduced. Furthermore, tetrahedra remain convex and planar.

During the extraction phase a single hyper-plane is intersected against the tetrahedral edges of the 4-D surface, producing a point set. This must be connected into a triangle mesh in order to construct a valid object. Triangulation must be efficient and produce a two-manifold polygon mesh. Fortunately, a two-manifold 4-D tetrahedral mesh will result in a two-manifold 3-D triangle mesh. There are either zero, three or four intersection points produced from each tetrahedron-hyperplane intersection. It is important that non-intersection be quickly detected (further discussed in section 4). Three intersection points can be triangulated immediately. All that remains is to determine the winding (triangle orientation). Edges in a tetrahedron are numbered in a consistent manner so that the order of their numbering matches the winding. This produces an intersection point set similarly ordered. The same approach applies to four-point intersections, which are split into two triangles. It essential to note that all points obtained from non-degenerate tetrahedral intersection lie on a convex hull, thereby making triangulation trivial.

4. CONSTANT EXTRUSION

An extrusion offset in the time dimension is required in animation to model spatial displacement through time. For example, a slanted space-time cylinder encodes a translation over time of the embedded circle. For topology alteration, however, constant extrusion suffices, producing a 4-D object rather than a space-time object. There are a number of possible improvements suggested by the resulting coherence.

Constant extrusion is implemented by embedding multiple copies of the 3-D object in 4-D, each with a constant offset in the higher dimension. The offset distant and number of embedded copies defines the height. This height directly impacts deformation costs and should be kept as small as possible (see section 7). The only calculation needed during offsetting is the derivation of the higher dimensional offset.

The vertex indices in a layer have the same order as previous layers and differ only by a constant offset. The resulting spatial coherence is exploited in the following cases:

- **Least Index Calculation** - Tetrahedral subdivision (as described in section 3) requires six compares to find the smallest vertex index in a triangular prism. But triangles in a lower layer will always contain lower indices since they occur first in the vertex array. This reduces the algorithm to three compares instead of six.
- **Generalisation of Structure** - Once one layer of the extruded mesh has been tetrahedronalised, all the other layers can be generalised to have the same tetrahedral structure. No additional calculations are needed to extrude beyond the first layer.

Sample points embedded in a deformable lattice for warping (see section 2), can be generalised from the first layer. The process is as follows: embed x, y, z co-ordinates of each vertex in the first layer. The corresponding vertices in subsequent layers will have the same x, y, z embedding. The w co-ordinate is identical for all vertices in a given layer and only needs to be calculated once per layer. Efficient vertex embedding has been actively researched [6] for virtual sculpting. Spatial coherence implies that vertex embedding for topology alteration in 4-D is only marginally more expensive than 3-D.

Extraction is performed by intersecting a hyper-plane with the 4-D object's edges. Constant extrusion allows the normal of this hyper-plane ($n = (0, 0, 0, 1)$) to remain unchanged for all extractions. The only variable is the hyper-plane origin. An extraction solution should efficiently pre-determine those edges intersected by the hyper-plane. It is trivial to determine this for an undeformed object: calculate which layer of the extrusion the hyper-plane is incident on; intersect tetrahedra edges in that layer. However, deformation needed to affect a topology change misaligns some edges with respect to the extrusion layer. Our solution is to calculate edge intersections that remain aligned to the layer after deformation, and then determine the subset of intersections that have become misaligned. This is much smaller and therefore is efficiently intersected. To determine the subset, we query the deformation algorithm for a list cells that underwent alteration.

5. LOCALISED TOPOLOGY ALTERATION

A topology change in a highly detailed mesh may only affect a small portion of it, making it counter-productive to extrude the whole mesh. Extruding only a localised 'portion of interest', as determined by user input, will produce a much smaller 4-D footprint, increasing overall efficiency..

The deformation stage is less costly because a smaller 4-D object has fewer vertices to embed and warp.

The extraction process for an undeformed tetrahedronalised prism produces four triangles. The union of these triangles is equivalent to the original embedding triangle (see Figure 4). The four extracted triangles are thus co-planar and have an identical convex hull to the original. It is obviously advantageous to ignore these four triangles in favour of the original. These triangles are identified and excluded from the whole extrusion, deformation and extraction process. Deformed prisms, however, must undergo projection, and thus a potential problem occurs when an undeformed triangle neighbours a set of deformed triangles: extraction results in a non two-manifold mesh (Figure 4). This is overcome by identifying the boundary edge between two deformed and one underformed triangle and connecting the new vertex to the apex of the neighbouring undeformed triangle.

6. PRELIMINARY RESULTS

We have implemented a prototype of four dimensional topology alteration with a tetrahedral representation and constant extrusion but without localised topology alteration. The following results were gathered on an Intel Xeon 1680 mhz processor with 2.56 gigabytes of ram for space-time objects with 15 levels and compare execution times (in milliseconds) with and without the constant extrusion optimizations.

Model	opt.	non-opt.	speedup
Sphere (1026 vertices)	211	420	×2.0
Cow (2904 vertices)	404	881	×2.1
Sphere (4098 vertices)	780	2701	×3.2
Sphere (16386 vertices)	2930	9820	×3.35

As indicated in our results there is already a substantial speedup. We expect further improvement once all our proposed enhancements have been fully implemented.

7. FUTURE WORK

Adaptive Refinement

The approximation quality of polygon meshes is degraded under deformation, specifically where curvature is altered. To compensate, a highly detailed space-time object can be used (effectively oversampling), by increasing the layers (and thus height) of extrusion. However, this impedes upon the efficiency of the topology alteration process. We seek to avoid this by using adaptive refinement. Efficient adaptive refinement has been researched for 3-D virtual sculpting [7], extending this process for 4-D will greatly increase the effectiveness of the topology alteration process.

Preventing Self Intersection

Self intersection, resulting from deformation, produces an incorrect representation and is not aesthetically pleasing. Furthermore, preventing self intersection for 4-D objects eliminates anomalies such as triangles with incorrect windings. Efficient self-intersection tests [8] have been developed for three-dimensional polygon meshes and these could be adapted to 4-D to provide efficient detection and prevention.

8. CONCLUSION

Figure 4: [Left] A triangle is extruded to define a prism which is subdivided into tetrahedra. The undeformed subdivided prism will produce four triangles filling the original shape. [Right] when deformed and undeformed triangles are neighbours, a two-manifold mesh is produced by connecting the new vertex to the apex of the undeformed triangle.

Topology alteration is vital if virtual sculpting is to successfully compete with traditional sculpting. We use spatial deformation, an interactive flexible modelling technique, to implement virtual sculpting but it does not change connectivity, a necessary condition for topological change. This limits deformation-based virtual sculpting to a subset of models that could be acquired by laser scanning a clay model.

In this paper, we propose a topology alteration system for spatial deformation that is appropriate to virtual sculping. Space-time objects are used to break the topology of an object with pleasing aesthetic results. Traditionally used for animation, space-time objects are less appropriate for modelling. We propose enhancements to allow for interactive feedback. We limit extrusion to a constant offset, thus producing a 4-D object and not a strict space-time object, and limit projection to the portion of the object undergoing topology alteration. The surface of the 4-D object is comprised of tetrahedra, resulting in a quick and minimal triangulation of the intersection point-set.

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