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Basic Infobase Change

Abstract. Generalisations of theory change involving arbitrary sets of wffs instead of belief sets have become known as base change. In one view, a base should be thought of as providing more structure to its generated belief set, and can be used to determine the theory change operation associated with a base change operation. In this paper we extend a proposal along these lines by Meyer et al. [15]. We take an infobase as a finite *sequence* of wffs, with each element in the sequence being seen as an independently obtained bit of information, and define appropriate infobase change operations. The associated theory change operations satisfy the AGM postulates for theory change [1]. Since an infobase change operation produces a new infobase, it allows for iterated infobase change. We measure iterated infobase change against the postulates proposed by Darwiche et al. [4, 5] and Lehmann [14].

Keywords: Theory change, belief revision, theory contraction, theory revision, base change, base contraction, base revision.

1. Introduction

It is generally accepted that belief sets do not have a rich enough structure to serve as appropriate models for epistemic states [11], [8]. This realisation has led some researchers to regard theory change as an elegant idealisation of a more general theory of belief change in which belief sets are replaced by arbitrary set of wffs known as bases.[1] The intuition is that some some of our beliefs have no independent standing, but arise only as beliefs derived from our more basic beliefs. And if our reason for believing such a derived belief disappears, then so should the belief.

One of the basic principles of base change is that it is sensitive to syntax. What is usually *not* made explicit, though, is that such an assertion can be interpreted in many ways. In the context of belief change, this sensitivity to syntax usually refers to one of the following two properties:

1. Belief bases offer a finer-grained approach than belief sets in the sense that two different belief bases may both be associated with the same belief set.
2. Belief change operations are interpreted on the symbol level and not on the knowledge level. In particular, this means that when performing a contracting operation on a base the resulting base must be a subset of the initial one.

[1] Although the original AGM postulates are not exclusively concerned with belief sets, the major results in [1] only hold for belief sets.

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In this paper we propose a form of belief change that attempts to retain the advantages of theory change as well as base change while simultaneously discarding the disadvantages of both. We suggest that a base should be thought of as providing more structure to its associated belief set. The added structure of the base can be used to pick an appropriate associated theory contraction operation, which, in turn, can be used to construct a range of suitable base contraction operations. Our proposal is an extension of the work of Meyer et al. [15] who regard an *infobase* as a finite set of wffs consisting of independently obtained bits of information. Taking AGM theory change [1] as the general framework in which to operate, they present a method that uses the structure of an infobase to determine which AGM theory change operation to associate with the infobase change operation to be constructed.

We improve on the proposal by Meyer et al. [15] in two ways. Firstly, and in line with the claim by Meyer et al. [15] that the definition of an infobase as a finite *set* of wffs is in conflict with the intuition of independently obtained wffs, we view an infobase as a finite *sequence* of wffs. This has a number of favourable consequences.[2] Secondly, the approach of Meyer et al. [15] associates a unique infobase contraction and revision operation with every infobase. We generalise this approach by allowing for a whole spectrum of infobase contraction and revision operations obtained from a given infobase, ranging from a "foundational" approach at one extreme to a "coherentist" approach at the other.

It is our contention that infobase change retains the advantages of (classic) base change in the sense that the structure of an infobase is used to guide the process of belief change. On the other hand, the process of performing infobase change is sufficiently semantic in nature to retain the advantages of theory change. In particular, we are able to obtain an appropriate version of Dalal's [3] principle of the irrelevance of syntax (cf. proposition 3.3) and, as can be seen in the discussion following example 2.16, infobase change is able to deal with a particularly vexing problem that has plagued various approaches to base change.

1.1. Preliminaries

For the rest of this paper L denotes any logical language, closed under the usual propositional connectives, and containing the symbols $\top$ and $\bot$. For every finite $C, D \subseteq L$ we write $C \diamond D$ as an abbreviation for $\{\gamma \diamond \delta \mid \gamma \in C$ and $\delta \in D\}$ where $\diamond \in \{\vee, \wedge\}$, $\neg C$ as an abbreviation for $\{\neg\gamma \mid \gamma \in C\}$, $\bigwedge C$ as an abbreviation for the conjunction of all elements in C, with $\bigwedge \emptyset = \top$,

[2] See [15] for a justification of this claim.

and $\bigvee C$ as an abbreviation for the disjunction of all elements in C, with $\bigvee \emptyset = \bot$. We assume L to have a two-valued model-theoretic semantics defining truth and falsity. The set of interpretations of L is denoted by U. We use $\Vdash$ for the relation from U to L denoting satisfaction and we assume that $\Vdash$ behaves classically with respect to the propositional connectives. We use $\top$ and $\bot$ as canonical representatives for the logically valid and logically invalid wffs respectively. For concreteness the reader may think of the logic under consideration as a (possibly infinitely generated) propositional logic. For every $X \subseteq L$, we denote the set of *models* of X by $M(X)$, and for $\alpha \in L$ we write $M(\alpha)$ instead of $M(\{\alpha\})$. Classical entailment (from $\wp L$ to L) is denoted by $\models$, and for $\alpha, \beta \in L$ we write $\alpha \models \beta$ instead of $\{\alpha\} \models \beta$. We also require $\models$ to satisfy *compactness*.[3] Closure under entailment is denoted by Cn. A *theory* or a *belief set* is a set $K \subseteq L$ closed under entailment. For every $V \subseteq U$, we let $\mathrm{Th}(V)$ denote the *theory determined by* V. A set $X \subseteq L$ *axiomatises* a set of interpretations V iff $\mathrm{Cn}(X) = \mathrm{Th}(V)$. For a set $X \subseteq L$, the*expansion* of X by a wff $\alpha \in L$ is defined as $X + \alpha = \mathrm{Cn}(X \cup \{\alpha\})$.

Our examples are phrased in propositional languages that are generated by at most three atoms. We use the letters p, q and r to denote these atoms, and interpretations of the languages will be represented by appropriate sequences of 0s and 1s, 0 representing falsity and 1 representing truth. The convention is that the first digit in the sequence represents the truth value of p, the second the truth value of q and the third the truth value of r.

An infobase will be represented as a finite sequence of wffs enclosed by square brackets. Although infobases are sensitive to the order in which wffs occur, as well as to their syntactical form, we shall see that these superficial qualities can be done away with by employing the notion of element-equivalence. Two infobases IB and IC are *element-equivalent*, written as $IB \approx IC$, iff for every β occurring in IB such that $\not\models \beta$, there is a unique (position-wise) logically equivalent wff γ occurring in IC, and for every γ occurring in IC such that $\not\models \gamma$, there is a unique (position-wise) logically equivalent wff β occurring in IB. We shall sometimes abuse notation slightly by applying the notion of element-equivalence to sets instead of infobases. For a finite sequence σ of wffs, we use the symbol $\bullet$ to denote concatenation by a single wff. The converse of concatenation (removing the last wff from a finite sequence σ) will be denoted by $\overleftarrow{\sigma}$. For a finite sequence σ of wffs, the *set* of wffs occurring in σ is denoted by $S(\sigma)$. That is, $S(\sigma) = \{\beta \mid \beta$ occurs in $\sigma\}$. We say that an infobase IB is *associated* with a belief set K (and K is *associated* with IB) iff $\mathrm{Cn}(S(IB)) = K$.

[3] That is, for every $X \subseteq L$, and every $\alpha \in L$, $X \models \alpha$, iff $X_F \models \alpha$ for some finite subset X_F of X.

Formally, we consider infobase change operations (which include contraction and revision operations) as functions from $\mathcal{IB} \times L$ to $\mathcal{IB}$, where $\mathcal{IB}$ is the set of all infobases. We shall also frequently assume the existence of a fixed infobase IB, and consider infobase IB-change operations as functions from L to $\mathcal{IB}$.

From results in [21], AGM theory change can be characterised by a set of total preorders (i.e. connected, reflexive, transitive relations) on U.[4] Let $\preceq$ be any total preorder on U. $x \in V \subseteq U$ is $\preceq$*-minimal* in V iff for every $y \in V$, $x \preceq y$. For a $V \subseteq U$, $\preceq$ is V*-smooth* iff for every $y \in V$ there is an $x \preceq y$ that is $\preceq$-minimal in V. $\preceq$ is *smooth* iff $\preceq$ is $M(\alpha)$-smooth for every $\alpha \in L$. We denote the set of $\preceq$-minimal elements of $M(\alpha)$ by $Min_{\preceq}(\alpha)$. Given an arbitrary set $X \subseteq L$, a preorder $\preceq$ on U is X*-faithful* iff $\preceq$ is smooth, $x \prec y$ for every $x \in M(X)$ and $y \notin M(X)$, and $x \preceq y$ for every $x, y \in M(X)$. The idea is to consider preorders in which the models of X, being the minimal, or "best" interpretations, are strictly below all other interpretations. The required results are obtained in terms of the following two identities:

(Def $-$ from $\preceq$) $\qquad K - \alpha = \mathrm{Th}(M(K) \cup Min_{\preceq}(\neg\alpha))$

(Def $*$ from $\preceq$) $\qquad K * \alpha = \mathrm{Th}(Min_{\preceq}(\alpha))$

THEOREM 1.1. 1. *Every K-faithful total preorder defines an* AGM *theory contraction using* (Def $-$ from $\preceq$). *Conversely, every* AGM *theory contraction can be defined in terms of a K-faithful total preorder using* (Def $-$ from $\preceq$).

2. *Every K-faithful total preorder defines an* AGM *theory revision using* (Def $*$ from $\preceq$). *Conversely, every* AGM *theory revision can be defined in terms of a K-faithful total preorder using* (Def $*$ from $\preceq$).

The following two identities can be used to define AGM theory revision and theory contraction in terms of one another.

(Harper Identity) $\qquad K - \phi = K \cap (K * \neg\phi)$

(Levi Identity) $\qquad K * \phi = (K - \neg\phi) + \phi$

2. Infobase change

Infobase change is similar in spirit to the knowledge level approach to base change favoured by [17]. The basic idea is to use the assumption of independence of the wffs in an infobase IB to construct the structures necessary

[4] This is similar to results in [9], [13] and [2].

for performing *theory* change. Both the current infobase and the obtained theory change operations are then used in the process of determining how to modify the existing infobase when confronted with new information, resulting in an operation which produces a new infobase from the current one.

To construct an infobase contraction, we first use the structure of the infobase IB to obtain an $S(IB)$-faithful total preorder. The theory contraction obtained from the $S(IB)$-faithful total preorder is taken to be the theory contraction associated with the infobase contraction that we aim to construct.

DEFINITION 2.1. For every infobase IB, a theory contraction $-$ is *associated with* an infobase IB-contraction $\ominus$ iff $\mathrm{Cn}(S(IB)) - \alpha = \mathrm{Cn}(S(IB \ominus \alpha))$ for every $\alpha \in L$.

Using the intuition associated with an infobase, we order the interpretations in U according to the number of wffs of IB they satisfy; the more they satisfy, the "better" they are deemed to be, and the lower down in the ordering they will be.

DEFINITION 2.2. For $u \in U$, we define u_{IB}, the *IB-number of u*, as the number of wffs β in IB such that $\nvDash \beta$ and $u \in M(\beta)$.

This ordering is used to obtain an appropriate $S(IB)$-faithful total preorder in terms of IB as follows:

(Def $\preceq$ from IB) $\quad u \preceq v$ iff $v_{IB} \leq u_{IB}$

DEFINITION 2.3. We refer to the faithful total preorder $\preceq_{IB}$ defined in terms of an infobase IB using (Def $\preceq$ from IB) as the *IB-induced* faithful total preorder.

The IB-induced faithful total preorder is used to construct a theory contraction as follows:

(Def $-_{IB}$ from IB) $\quad \mathrm{Cn}(S(IB)) -_{IB} \alpha = \mathrm{Th}(M(S(IB)) \cup Min_{\preceq_{IB}}(\neg\alpha))$

DEFINITION 2.4. The theory contraction $-_{IB}$ defined in terms of an infobase IB using (Def $-_{IB}$ from IB) is referred to as the *IB-induced theory contraction.*

Clearly the IB-induced theory contraction is an AGM theory contraction. Associating the IB-induced theory contraction with the infobase IB-contraction allows us to determine which wffs in IB should be retained, and which cannot be retained, after a contraction of IB.

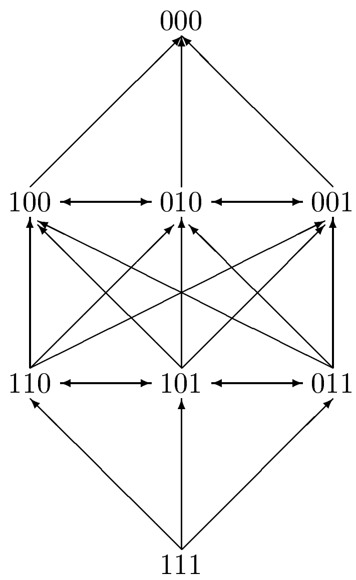

Figure 1. A graphical representation of the IB-induced faithful total preorder $\preceq_{IB}$, with $IB = [p, q, r]$. For every $u, v \in U$, $u \preceq_{IB} v$ iff (u, v) is in the reflexive transitive closure of the relation determined by the arrows. Interpretations are represented as ordered triples of 0s and 1s, 0 representing falsity and 1 representing truth. The first digit in a triple represents the truth value of p, the second the truth value of q and the third the truth value of r.

DEFINITION 2.5. The set of *α-discarded wffs* (of an infobase IB) is defined as $IB^{-\alpha} = \{\beta \in S(IB) \mid \beta \notin \text{Cn}(S(IB)) -_{IB} \alpha\}$. We refer to $S(IB) \setminus IB^{-\alpha}$ as the set of *α-retained* wffs (of IB).

The α-retained wffs are precisely the wffs in IB that should be retained when contracting IB by α, while the α-discarded wffs are replaced with appropriately weakened wffs. In deciding on an appropriate method for the weakening of the α-discarded wffs, it is necessary to strike the right balance between what we tentatively refer to as a *coherentist* approach, emphasising knowledge level matters, and a *foundationalist* approach, emphasising the independence of the wffs occurring in IB. The following example serves to make these matters concrete.

EXAMPLE 2.6. Consider the infobase $IB = [p, q, r]$. Figure 1 gives a graphical representation of the IB-induced faithful total preorder $\preceq_{IB}$. The wffs p, q and r each represents independently obtained information. So, when contracting IB by $p \wedge q$, the resulting infobase should contain weakened versions of the two $(p \wedge q)$-discarded wffs p and q, and should contain the $(p \wedge q)$-retained wff r itself. But what should the weakened versions of p

and q look like? An application of the coherentist approach on a local level suggests that, in order to minimise the loss of information, one should add only the minimal models of $\neg(p \wedge q)$ to the models of both p and q, and let the corresponding wffs be the appropriate weakened versions. The weakened version of p would be logically equivalent to $p \vee (q \wedge r)$ and the weakened version of q would be logically equivalent to $q \vee (p \wedge r)$. On the other hand, the foundationalist approach, which stresses the independence of the wffs in IB, suggests that the presence of r should have no effect on the weakened versions of p and q. In this view, the wff $p \vee q$ (or any wff logically equivalent to it) would be a suitable choice for the weakened versions of both p and q.[5]

■

There does not seem to be a definite answer to the question of which one of these two approaches to infobase change is the "correct" one. They should rather be seen as opposites on a whole spectrum of possibilities. The coherentist approach can be described as the case where *all* the wffs in IB play a role in determining the weakened versions of the α-discarded wffs, while the foundationalist approach ensures that only the set of α-discarded wffs themselves is involved in the construction of their weakened versions. Given these two opposites, it also seems perfectly reasonable to allow for *any* set of wffs in between (i.e., containing the α-discarded wffs and included in $S(IB)$) to be involved in the construction of the weakened versions of the α-discarded wffs.

DEFINITION 2.7. Given an infobase IB and a wff α, a set R is said to be (IB, α)*-relevant* iff $IB^{-\alpha} \subseteq R \subseteq S(IB)$.

Our goal is to ensure that, in the process of obtaining the weakened versions of the α-discarded wffs, the effect of the wffs *not* in the (IB, α)-relevant set R are neutralised. To do so, we should not just add the $\preceq_{IB}$-minimal models of $\neg\alpha$, but also any other models of $\neg\alpha$ that behave exactly like the $\preceq_{IB}$-minimal models with respect to the wffs in R, but that might differ from the $\preceq_{IB}$-minimal models on the truth value of the wffs in $S(IB) \setminus R$.

DEFINITION 2.8. For $X \subseteq L$ and $u, v \in U$, u is X*-equivalent* to v, written $u \equiv_X v$, iff for every $\chi \in X$, $u \in M(\chi)$ iff $v \in M(\chi)$.

Observe that, for the $(IB, p \wedge q)$-relevant set $R = \{p, q\}$ in example 2.6, it follows that 100 and 010 are R-equivalent to the minimal models 101 and

[5] Keep in mind, though, that, regardless of which of these approaches to infobase change are used, the theory contractions associated with these different infobase IB-contractions will be identical, since we associate the IB-induced theory contraction with every possible infobase IB-contraction.

011 respectively, and adding them to the models of p (and q) as well results in weakened versions of p and q that are logically equivalent to $p \vee q$, which is in line with the foundationalist intuition described above.

In general, we obtain the weakened version of every α-discarded wff β as follows. We need some appropriate set of interpretations that can be added to the models of β to obtain the set of models of its weakened version. Once we have decided on an (IB, α)-relevant set R, we use the set of minimal models of $\neg\alpha$ as our starting point and then try to expand it so that only elements in R have any influence, thus neutralising the possible influence of any of remaining wffs in IB. This is accomplished by including all the models of $\neg\alpha$ that are R-equivalent to some minimal model of $\neg\alpha$.

DEFINITION 2.9. Let R be any (IB, α)-relevant set. For $u \in Min_{\preceq_{IB}}(\neg\alpha)$, let

$$\begin{aligned} N_u^R(\neg\alpha) &= \{v \in M(\neg\alpha) \mid v \equiv_R u\}, \\ N_{IB}^R(\neg\alpha) &= \textstyle\bigcup_{u \in Min_{\preceq_{IB}}(\neg\alpha)} N_u^R(\neg\alpha). \end{aligned}$$

We refer to $N_{IB}^R(\neg\alpha)$ as the (R, α)-*neutralised models* of IB.

We take the (R, α)-neutralised models as the set of interpretations to be added to the models of each α-discarded wff. We can think of the (R, α)-neutralised models as a set of interpretations in which the influence of the wffs not in R has been removed, but in which the wffs in R have the same impact as on the minimal models of $\neg\alpha$. To summarise, we intend to obtain the infobase resulting from an α-contraction of the infobase IB by weakening the α-discarded wffs in the manner described above, and keeping the α-retained wffs as they are.

It turns out that there is an elegant way to provide a uniform description of infobase contraction. We can describe it as a process in which *all* the wffs in the current infobase are replaced with weaker versions, but where the "weaker" version of every α-retained wff turns out to be logically equivalent to the wff itself.

DEFINITION 2.10. Let R be any (IB, α)-relevant set. For every $\beta \in S(IB)$, we let $N_\beta^R(\neg\alpha) = \bigcup_{u \in Min_{\preceq_{IB}}(\neg\alpha) \setminus M(\beta)} N_u^R(\neg\alpha)$. We refer to $N_\beta^R(\neg\alpha)$ as the $(R, \alpha\beta)$-*neutralised models* of IB.

The next proposition shows that an α-retained wff β has no (R, α, β)-neutralised models, and that, for an α-discarded wff β, adding the (R, α, β)-neutralised models to the models of β, has the same effect as adding the (R, α)-neutralised models.

PROPOSITION 2.11. *Let R be any (IB, α)-relevant set.*

1. *If $\beta \in S(IB) \setminus IB^{-\alpha}$ then $N_\beta^R(\neg\alpha) = \emptyset$.*
2. *If $\beta \in IB^{-\alpha}$ then $M(\beta) \cup N_\beta^R(\neg\alpha) = M(\beta) \cup N_{IB}^R(\neg\alpha)$.*

PROOF. 1. Suppose that $\beta \in S(IB) \setminus IB^{-\alpha}$. Then $\beta \in \mathrm{Cn}(S(IB)) -_{IB} \alpha$ and thus $Min_{\preceq_{IB}}(\neg\alpha) \subseteq M(\beta)$. And therefore

$$N_\beta^R(\neg\alpha) = \bigcup_{u \in Min_{\preceq_{IB}}(\neg\alpha) \setminus M(\beta)} N_u^R(\neg\alpha) = \emptyset.$$

2. Suppose that $\beta \in IB^{-\alpha}$. The left-to-right inclusion is immediate. For the right-to-left inclusion we have to show that $\bigcup_{u \in Min_{\preceq_{IB}}(\neg\alpha) \cap M(\beta)} N_u^R(\neg\alpha) \subseteq M(\beta)$.

So pick any $u \in Min_{\preceq_{IB}}(\neg\alpha) \cap M(\beta)$ and $v \in N_u^R(\neg\alpha)$. Then $v \equiv_R u$ and since $\beta \in R$, it follows that $v \in M(\beta)$. ■

Proposition 2.11 allows us to describe an α-contraction of an infobase IB by adding to the models of a wff β in IB, the set $N_\beta^R(\neg\alpha)$, and replacing β with an axiomatisation of this set of interpretations. Of course, such a description only makes sense if these sets of interpretations can be axiomatised by single wffs. While this is immediate for the finitely generated propositional logics, the next result shows that it also holds in the more general case.

DEFINITION 2.12. Let R be any (IB, α)-relevant set, and for $\beta \in S(IB)$, let IB_β^α be the set containing every ordered subsequence C of IB such that $|C| = u_{IB}$ for some $u \in (Min_{\preceq_{IB}}(\neg\alpha) \cap M(S(C))) \setminus M(\beta)$ (where u_{IB} is the IB-number of u). We define the *α-weakened version* of β, with respect to R, as

$$w_{(IB,\alpha)}^R(\beta) = \beta \vee \Big(\bigvee_{C \in IB_\beta^\alpha} \Big(\Big(\bigwedge (S(C) \setminus (S(IB) \setminus R)) \Big) \wedge \Big(\bigwedge \neg(R \setminus S(C)) \Big) \wedge \neg\alpha \Big) \Big).$$

PROPOSITION 2.13. *Let R be an (IB, α)-relevant set. For every $\alpha \in L$ and every $\beta \in S(IB)$, $M(w_{(IB,\alpha)}^R(\beta)) = M(\beta) \cup N_\beta^R(\neg\alpha)$.*

PROOF. Define IB_β^α as in definition 2.12. If $IB_\beta^\alpha = \emptyset$ then it follows easily that $Min_{\preceq_{IB}}(\neg\alpha) \setminus M(\beta) = \emptyset$, which means that $Min_{\preceq_{IB}}(\neg\alpha) \subseteq M(\beta)$ and therefore that $N_\beta^R(\neg\alpha) = \emptyset$. So we only need to consider the case where $IB_\beta^\alpha \neq \emptyset$. Then every $u \in Min_{\preceq_{IB}}(\neg\alpha) \setminus M(\beta)$ is a model of $S(C)$ for some $C \in IB_\beta^\alpha$. Pick any $C \in IB_\beta^\alpha$ and any $u \in (Min_{\preceq_{IB}}(\neg\alpha) \cap M(S(C))) \setminus$

$M(\beta)$. Observe that every model of $S(C) \cup \{\neg\alpha\}$ is a $\preceq_{IB}$-minimal element of $M(\neg\alpha)$, which ensures that every element of $(R \setminus S(C)) \setminus \mathrm{Cn}(\top)$ is false in all the models of $S(C) \cup \{\neg\alpha\}$. We record this result formally.

$$\forall\gamma \in (R \setminus S(C)) \setminus \mathrm{Cn}(\top),\ \ \forall v \in M(S(C) \cup \{\neg\alpha\}),\ \ v \notin M(\gamma) \qquad (1)$$

We show that $M((S(C) \setminus (S(IB) \setminus R)) \cup \neg((R \setminus S(C)) \setminus \mathrm{Cn}(\top)) \cup \{\neg\alpha\}) = N_u^R(\neg\alpha)$. From (1) it follows that $u \notin M(\gamma)$ for every $\gamma \in (R \setminus S(C)) \setminus \mathrm{Cn}(\top)$ and therefore that

$$u \in M\left((S(C) \setminus (S(IB) \setminus R)) \cup \neg((R \setminus S(C)) \setminus \mathrm{Cn}(\top)) \cup \{\neg\alpha\}\right).$$

Now pick any $v \in M((S(C) \setminus (S(IB) \setminus R)) \cup \neg((R \setminus S(C)) \setminus \mathrm{Cn}(\top)) \cup \{\neg\alpha\})$ and any $\rho \in R$. We only consider the case where $\rho \not\equiv \top$. If $\rho \in S(C)$ then clearly $u \in M(\rho)$ iff $v \in M(\rho)$, so suppose $\rho \notin S(C)$. Then by (1) again, $u \notin M(\rho)$. Furthermore, since $v \in M(\neg((R \setminus S(C)) \setminus \mathrm{Cn}(\top)))$, it follows that $v \notin M(\rho)$ and thus that $u \in M(\rho)$ iff $v \in M(\rho)$. Finally, it is clear that $v \in M(\neg\alpha)$. We have thus shown that $v \in N_u^R(\neg\alpha)$. Conversely, pick any $v \in N_u^R(\neg\alpha)$. Clearly $v \in M(\neg\alpha)$, and since $u \in M((S(C) \setminus (S(IB) \setminus R)) \cup \neg((R \setminus S(C)) \setminus \mathrm{Cn}(\top)) \cup \{\neg\alpha\})$, so is v.

It is clear that $M((S(C) \setminus (S(IB) \setminus R)) \cup \neg((R \setminus S(C)) \setminus \mathrm{Cn}(\top)) \cup \{\neg\alpha\})$ is axiomatised by the wff

$$(\neg\alpha)_C^R = \left(\bigwedge (S(C) \setminus (S(IB) \setminus R))\right) \wedge \left(\bigwedge \neg((R \setminus S(C)) \setminus \mathrm{Cn}(\top))\right) \wedge \neg\alpha$$

and it thus follows that $M((\neg\alpha)_C^R) = N_u^R(\neg\alpha)$. So we have shown that if $IB_\beta^\alpha \neq \emptyset$, then

$$\forall C \in IB_\beta^\alpha, \exists u \in (Min_{\preceq_{IB}}(\neg\alpha) \cap M(S(C))) \setminus M(\beta) \text{ and} \qquad (2)$$

$$\forall C \in IB_\beta^\alpha, \forall u \in (Min_{\preceq_{IB}}(\neg\alpha) \cap M(S(C))) \setminus M(\beta),$$
$$M\left((\neg\alpha)_C^R\right) = N_u^R(\neg\alpha). \qquad (3)$$

We now show that $N_\beta^R(\alpha) = M(\bigvee_{C \in IB_\beta^\alpha} (\neg\alpha)_C^R)$, from which the required result follows. Pick a $v \in N_\beta^R(\neg\alpha)$. There is a $u \in Min_{\preceq_{IB}}(\neg\alpha) \setminus M(\beta)$ such that $v \in N_u^R(\neg\alpha)$, and by (3) it follows that for some $C \in IB_\beta^\alpha$, $v \in N_u^R(\neg\alpha) = M\left((\neg\alpha)_C^R\right)$. So clearly $v \in M\left(\bigvee_{C \in IB_\beta^\alpha} (\neg\alpha)_C^R\right)$. Conversely, pick any $v \in M\left(\bigvee_{C \in IB_\beta^\alpha} (\neg\alpha)_C^R\right)$. Then v is a model of $(\neg\alpha)_C^R$ for some $C \in IB_\beta^\alpha$. By (2) there is a $u \in (Min_{\preceq_{IB}}(\neg\alpha) \cap M(S(C))) \setminus M(\beta)$, and by (3), $N_u^R(\neg\alpha) = M\left((\neg\alpha)_C^R\right)$. So $v \in N_u^R(\neg\alpha)$ and thus $v \in N_\beta^R(\neg\alpha)$. ∎

We are now almost in a position to define *basic* infobase contraction.

DEFINITION 2.14. A function $rs: \mathcal{IB} \times \wp L \to \wp\wp L$ is a *relevance selection* function iff
1. $IB^{-\alpha} \subseteq rs(IB, \alpha) \subseteq IB$,
2 if $\alpha \equiv \beta$ then $rs(IB, \alpha) = rs(IB, \beta)$, and
3. if $IB \approx IC$ then $rs(IB, \alpha) \approx rs(IC, \alpha)$.

Intuitively, a relevance selection function indicates which of the wffs in IB should play a role in determining the weakened versions during a contraction. Observe that $rs(IB, \alpha)$ is (IB, α)-relevant.

DEFINITION 2.15. 1. An infobase change operation $\ominus$ is a *basic infobase contraction* iff there is a relevance selection function rs such that, for every $IB \in \mathcal{IB}$ and every $\alpha \in L$, $IB \ominus \alpha$ is obtained by replacing every wff β in IB with $w_{(IB,\alpha)}^{rs(IB,\alpha)}(\beta)$, the α-weakened version of β with respect to $rs(IB, \alpha)$.

2. For every $IB \in \mathcal{IB}$, an infobase IB-change operation $\ominus_{IB}$ is a *basic infobase IB-contraction* iff it can be obtained from a basic infobase contraction $\ominus$ by fixing the infobase IB. That is, iff $IB \ominus_{IB} \alpha = IB \ominus \alpha$ for every $\alpha \in L$.

In practice an agent would, when contracting by a wff α, specify which of the wffs in an infobase IB it regards as being relevant to the contraction.[6] The appropriate relevance selection function would then be determined by taking the union of $IB^{-\alpha}$ and the set of wffs deemed to be relevant by the agent. This can be justified as follows: By the very nature of infobase contraction the wffs in $IB^{-\alpha}$ *have* to be relevant when contracting by α. Whether any of the remaining wffs are relevant is a matter to be determined by extra-logical information to be supplied by the agent.

We conclude this section with an example illustrating the partial construction of some basic infobase contractions.

EXAMPLE 2.16. Let $IB = [p, q]$. Figure 2 contains a graphical representation of the IB-induced faithful total preorder $\preceq_{IB}$. Then

$$\mathrm{Cn}(S(IB)) -_{IB} p = \mathrm{Cn}(q), \ \ IB^{-p} = \{p\}$$
$$IB_p^p = \{[q]\}, \ \ IB_q^p = \emptyset,$$
$$\mathrm{Cn}(S(IB)) -_{IB} (p \wedge q) = \mathrm{Cn}(p \vee q)$$
$$IB^{-(p\wedge q)} = \{p, q\}, \ \ IB_p^{p\wedge q} = \{[q]\}, \text{ and } IB_q^{p\wedge q} = \{[p]\}.$$

[6] *How* an agent would determine which wffs are relevant is beyond the scope of this paper.

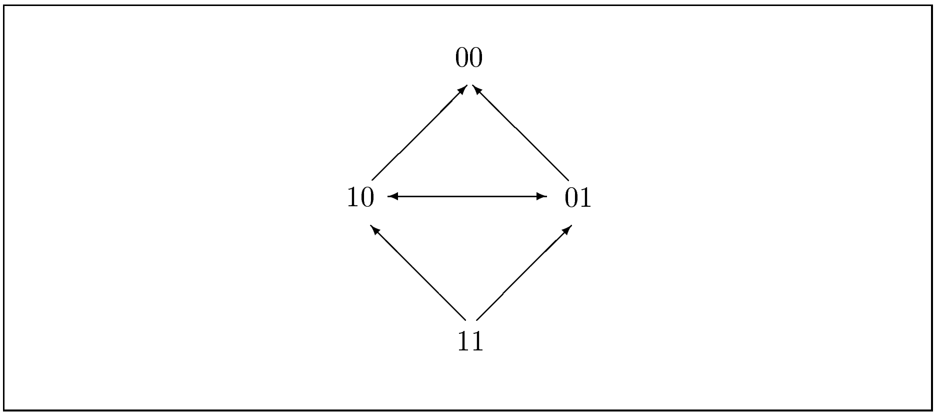

Figure 2. A graphical representation of the IB-induced faithful total preorder $\preceq_{IB}$, with $IB = [p, q]$. For every $u, v \in U$, $u \preceq_{IB} v$ iff (u, v) is in the reflexive transitive closure of the relation determined by the arrows.

Now observe that $w^{IB^{-p}}_{(IB,p)}(p) = p \vee (\top \wedge \neg p \wedge \neg p) \equiv \top$ and that $w^{IB^{-p}}_{(IB,p)}(q) = q \vee \bot \equiv q$. Furthermore, since $S(IB) = B^{-p\wedge q}$, note that

$$\begin{aligned} w^{S(IB)}_{(IB,p\wedge q)}(p) &= w^{IB^{-p\wedge q}}_{(IB,p\wedge q)}(p) = p \vee (q \wedge \neg p \wedge \neg(p \wedge q)) \text{ and} \\ w^{S(IB)}_{(IB,p\wedge q)}(q) &= w^{IB^{-p\wedge q}}_{(IB,p\wedge q)}(q) = q \vee (p \wedge \neg q \wedge \neg(p \wedge q)) . \end{aligned}$$

It can be verified that both $w^{S(IB)}_{(IB,p\wedge q)}(p)$ and $w^{S(IB)}_{(IB,p\wedge q)}(q)$ are logically equivalent to $p \vee q$. There is thus at least one basic infobase contraction $\ominus$ such that

$$IB \ominus p = [w^{IB^{-p}}_{(IB,p)}(p), w^{IB^{-p}}_{(IB,p)}(q)] \approx [\top, q]$$

and

$$IB \ominus (p \wedge q) = [w^{IB^{-p\wedge q}}_{(IB,p\wedge q)}(p), w^{IB^{-p\wedge q}}_{(IB,p\wedge q)}(q)] \approx [p \vee q, p \vee q].$$

Furthermore, observe that $w^{S(IB)}_{(IB,p)}(p) = p \vee (q \wedge \neg p \wedge \neg p) \equiv p \vee q$ and that $w^{S(IB)}_{(IB,p)}(q) = q \vee \bot \equiv q$. So there is least one infobase contraction $\ominus'$ such that

$$IB \ominus' p = \left[w^{S(IB)}_{(IB,p)}(p), w^{S(IB)}_{(IB,p)}(q)\right] \approx [p \vee q, q]$$

and

$$IB \ominus' (p \wedge q) = \left[w^{S(IB)}_{(IB,p\wedge q)}(p), w^{S(IB)}_{(IB,p\wedge q)}(q)\right] \approx [p \vee q, p \vee q]. \qquad \blacksquare$$

One of the main differences between infobase change and many approaches to base change is illustrated by the last part of example 2.16, where a wff that is not contained in the infobase $IB = [p, q]$ finds its way into the resulting infobase $IB \ominus' p \wedge q$. It is with this kind of example in mind that Rott [22] writes as follows (In the quotation H represents the base $\{p, q\}$):

> Even after conceding that one of p and q may be false, we should still cling to the belief that the other one is true. But $H' = \{p \vee q\}$ is no base which can be constructed naturally from H — it certainly does not record any explicit belief. We are faced with a deep-seated dilemma ...

Rott ultimately decides against the inclusion of such wffs, arguing that bases should only contain explicit beliefs.[7]

A careful analysis shows that the way in which infobase change solves this problem — replacing wffs to be removed with appropriately weakened ones — comes about because infobase change is sufficiently semantic in nature.

3. Properties of basic infobase change

Infobase change is, in some respects, similar in spirit to the base change proposals of Nebel [17] and Nayak [16]. Since the initial infobase proposal by Meyer et al. [15] contains a detailed comparison of infobase change with the work of Nebel and Nayak, we shall not repeat it here.

Given the intuition associated with infobase change, it is to be expected that the IB-induced theory contraction is the theory contraction associated with every basic infobase IB-contraction. We first present a preliminary result, indicating that for every (IB, α)-relevant set R, the models of the α-retained wffs that are also (R, α)-neutralised models, are precisely the $\preceq_{IB}$-minimal models of $\neg\alpha$.

LEMMA 3.1. *If R is an (IB, α)-relevant set, then*

$$N^R_{IB}(\neg\alpha) \cap M(S(IB) \setminus IB^{-\alpha}) = Min_{\preceq_{IB}}(\neg\alpha).$$

PROOF. By definition, $S(IB) \setminus IB^{-\alpha} \subseteq \mathrm{Cn}(S(IB)) -_{IB} \alpha$ and thus

$$M(S(IB)) \cup Min_{\preceq_{IB}}(\neg\alpha) \subseteq M(S(IB) \setminus IB^{-\alpha}).$$

Furthermore, $Min_{\preceq_{IB}}(\neg\alpha) \subseteq N^R_{IB}(\neg\alpha)$, and so $Min_{\preceq_{IB}}(\neg\alpha) \subseteq N^R_{IB}(\neg\alpha) \cap M(S(IB) \setminus IB^{-\alpha})$. Conversely, pick any $v \in N^R_{IB}(\neg\alpha) \cap M(S(IB) \setminus IB^{-\alpha})$. That is, v satisfies all the α-retained wffs, v is a model of $\neg\alpha$ and there is a $\preceq_{IB}$-minimal model u of $\neg\alpha$ that satisfies exactly the same wffs in R as v does (which includes the α-discarded wffs). Because $u \in Min_{\preceq_{IB}}(\neg\alpha)$, it

[7] Hansson [12] mentions the use of disjunctively closed bases (in which the disjunction $\alpha \vee \beta$ of every $\alpha, \beta \in B$ is also in B) as a possible solution to problems of this kind. Unfortunately this ensures that bases can't be finite. And in any case, Hansson does not regard it as an acceptable solution, warning that it should be seen as an interesting special case, rather than a required property of bases.

follows from the definition of $-_{IB}$ and $IB^{-\alpha}$ that u also satisfies all the wffs in $S(IB) \setminus IB^{-\alpha}$. So u and v satisfy exactly the same wffs occurring in IB, which means that $v \in Min_{\preceq_{IB}}(\neg\alpha)$. ■

The result above is used to prove that the IB-induced contraction $-_{IB}$ is the theory contraction associated with every basic infobase IB-contraction.

PROPOSITION 3.2. *Let $\ominus$ be any basic infobase contraction. Then*

$$\mathrm{Cn}(S(IB)) -_{IB} \alpha = \mathrm{Cn}(S(IB \ominus \alpha)).$$

PROOF. Let rs be the relevance selection function used to define $\ominus$. By propositions 2.11 and 2.13,

$$\begin{aligned} M(S(IB \ominus \alpha)) &= \Big[\bigcap_{\beta \in IB^{-\alpha}} \Big(M(\beta) \cup N_{IB}^{rs(IB,\alpha)}(\neg\alpha) \Big) \Big] \cap M(S(IB) \setminus IB^{-\alpha}) \\ &= \Big[\Big(\bigcap_{\beta \in IB^{-\alpha}} M(\beta) \Big) \cup N_{IB}^{rs(IB,\alpha)}(\neg\alpha) \Big] \cap M(S(IB) \setminus IB^{-\alpha}) \\ &= \Big(M(IB^{-\alpha}) \cup N_{IB}^{rs(IB,\alpha)}(\neg\alpha) \Big) \cap M(S(IB) \setminus IB^{-\alpha}) \\ &= M(S(IB)) \cup \Big(N_{IB}^{rs(IB,\alpha)}(\neg\alpha) \cap M(S(IB) \setminus IB^{-\alpha}) \Big) \\ &= M(S(IB)) \cup Min_{\preceq_{IB}}(\neg\alpha) \text{ by lemma 3.1,} \end{aligned}$$

and thus $\mathrm{Cn}(S(IB)) -_{IB} \alpha = \mathrm{Cn}(S(IB \ominus \alpha))$. ■

Since two different infobases, say IB and IC, associated with the same theory may induce different total preorders on U, it follows that the theory contractions associated with the infobase IB- and IC-contractions may differ. In this sense, then, infobase change is sensitive to syntax. This kind of syntax-sensitivity is fairly common among approaches to base change. Indeed, it is usually part of the motivation for moving from theory change to base change. Unfortunately it usually implies a (less desirable) form of syntax-sensitivity as well, where different, but logically equivalent, wffs in a base may lead to different results. The significance of the next proposition is that it shows that infobase change does not suffer from the latter form of syntax-sensitivity. In particular, the syntactic form of the wffs in an infobase, as well as the form of the wff with which to contract, are irrelevant. The result can thus be seen as an appropriate version of Dalal's [3] principle of the irrelevance of syntax.

PROPOSITION 3.3. *Let $\ominus$ be a basic infobase contraction, and suppose that $IB \approx IC$ and $\beta \equiv \gamma$. Then $IB \ominus \beta \approx IC \ominus \gamma$.*

PROOF. Let rs be the relevance selection function used to obtain $\ominus$. Since IB and IC are element-equivalent, $u_{IB} = u_{IC}$ for every $u \in U$, and so the IB-induced faithful total preorder is exactly the same as the IC-induced faithful preorder. By the properties of a relevance selection function, it then follows that $N_{IB}^{rs(IB,\beta)}(\neg\beta) = N_{IC}^{rs(IC,\gamma)}(\neg\gamma)$. So, by propositions 2.11 and 2.13, $w_{(IB,\beta)}^{IB}(\beta') \equiv w_{(IC,\gamma)}^{IC}(\gamma')$ for every β' in IB and every γ' in IC such that $\beta' \equiv \gamma'$, from which the required result follows. ■

In short, while infobase change exploits the syntactic structure of an infobase, it retains a semantic dimension which is lacking in other, more syntactically-oriented, forms of base change.

3.1. Infobase contraction and reason maintenance

In the context of infobase change, reason maintenance [6] amounts to ensuring that the contraction of IB by a wff α in IB results in the removal of all the wffs that are dependent on α for being in $\mathrm{Cn}(S(IB))$. Fuhrmann [7] has given a precise meaning to the idea of a wff being dependent on α (for being in $\mathrm{Cn}(S(IB))$).[8]

DEFINITION 3.4. A wff $\beta \in L$ is *IB-dependent* on α iff $\alpha \in S(IB)$ and $\beta \in \mathrm{Cn}(S(IB))$, but $\beta \notin \mathrm{Cn}(S(IB) \setminus \{\alpha\})$.

The next result shows that basic infobase contraction incorporates reason maintenance.

PROPOSITION 3.5. *Let $\ominus$ be a basic infobase contraction. If β is IB-dependent on α then $\beta \notin \mathrm{Cn}(S(IB \ominus \alpha))$.*

PROOF. Since $\beta \in \mathrm{Cn}(S(IB))$, but $\beta \notin \mathrm{Cn}(S(IB) \setminus \{\alpha\})$, there has to be a model u of $S(IB) \setminus \{\alpha\}$ in which both α and β are false. So $u \in M(\neg\alpha)$ and $u \notin M(S(IB))$. Now, there is only one wff in IB, namely α, that is false in u (although IB may contain multiple instances of α). So any interpretation v for which $v_{IB} > u_{IB}$, has to be a model of $S(IB)$ and hence of α. Therefore $u \in Min_{\preceq_{IB}}(\neg\alpha)$, and because $u \notin M(\beta)$, it follows that $\beta \notin \mathrm{Cn}(S(IB)) -_{IB} \alpha$. So $\beta \notin \mathrm{Cn}(S(IB \ominus \alpha))$ by proposition 3.2. ■

Of course, the contraction of IB by a wff α in IB is not the only way to remove α from the infobase IB. In the light of this, it seems reasonable to inquire whether the wffs that are IB-dependent on α will also be discarded

[8] Fuhrmann works with belief bases and not infobases, and our definition of IB-dependence is thus a slight generalisation of the notion he defines.

if α is discarded during the contraction of IB by some wff other than α itself. That is, if α is in IB and $\alpha \notin \text{Cn}(S(IB \ominus \gamma))$, will it be the case that $\beta \notin \text{Cn}(S(IB \ominus \gamma))$ for every β that is IB-dependent on α? This property is known as Fuhrmann's [7] *filtering condition.* It is easy to see that basic infobase contraction can violate the filtering condition. For example, it is readily verified that for *any* basic infobase contraction, the contraction of the infobase $IB = [p \wedge q]$ by p results in an infobase in which $p \wedge q$ is replaced by the wff $w^{S(IB)}_{(IB,p)}(p \wedge q)$ which is logically equivalent to $p \rightarrow q$. And since $w^{S(IB)}_{(IB,p)}(p \wedge q)$ is clearly IB-dependent on $p \wedge q$, the filtering condition is violated. But such a violation is to be expected. Given the intuition associated with infobases, the filtering condition is clearly too strong a requirement to impose. For the filtering condition requires that for any infobase contraction $\ominus$, $\text{Cn}(S(IB \ominus \gamma)) = \text{Cn}(\top)$ for any singleton infobase IB, and any $\gamma \in \text{Cn}(S(IB))$ (where $\nvDash \gamma$), thus leaving no room for weakening the wff in IB to anything but a logically valid wff.

3.2. Infobase revision

Basic infobase revision is defined by an appeal to the following infobase analogue of the Levi Identity:

(Def $\circledast$ from $\ominus$) $\qquad IB \circledast \alpha = (IB \ominus \neg\alpha) \bullet \alpha$

Definition 3.6. An infobase change operation $\circledast$ is a *basic infobase revision* iff it can be defined in terms of a basic infobase contraction $\ominus$ using (Def $\circledast$ from $\ominus$).

Given this connection, it is to be expected that basic infobase revision satisfies properties that are similar to those proved in sections 3 and 3.1. The next corollary shows that this is indeed the case.

Definition 3.7. A theory revision $*$ is *associated with* an infobase IB-revision $\circledast$ (for some infobase IB) iff $\text{Cn}(IB) * \alpha = \text{Cn}(IB \circledast \alpha)$ for every $\alpha \in L$.

(Def $*_{IB}$ from IB) $\qquad \text{Cn}(S(B)) *_{IB} \alpha = \text{Th}(Min_{\preceq_{IB}}(\alpha))$

Definition 3.8. The theory revision $*_{IB}$ defined in terms of an infobase IB using (Def $*_{IB}$ from IB) is referred to as the *IB-induced theory revision.*

From theorem 1.1 it follows that the IB-induced theory revision is an AGM theory revision.

COROLLARY 3.9. *Let $\ominus$ be a basic infobase contraction, and let $\circledast$ be the infobase revision defined in terms of $\ominus$ using* (Def $\circledast$ from $\ominus$).

1. *If $IB \approx IC$ and $\alpha \equiv \beta$ then $IB \circledast \alpha \approx IC \circledast \beta$.*
2. $\mathrm{Cn}(S(IB \circledast \alpha)) = \mathrm{Cn}(S(IB)) *_{IB} \alpha$.
3. *If β is IB-dependent on α, then $\beta \notin \mathrm{Cn}(S(B \circledast \neg\alpha))$.*

PROOF. 1. Follows from proposition 3.3.

2. Follows from proposition 3.2, by noting that $Min_{\preceq_{IB}}(\alpha) \subseteq M(S(IB))$ if $\neg\alpha \notin \mathrm{Cn}(S(IB))$, and by recalling that $\mathrm{Cn}(S(IB)) *_{IB} \alpha = \mathrm{Th}(Min_{\preceq_{IB}}(\alpha))$.

3. Follows from part (2) of this corollary, and by an argument similar to the proof of proposition 3.5. ■

Part 1 of corollary 3.9 shows that basic infobase revision is insensitive to the syntactic form of the wffs in an infobase, as well as to the syntactic form of the wff with which to revise, part 2 shows that the theory revision associated with a basic infobase revision is the IB-induced revision function, and part 3 shows that basic infobase revision can be said to perform reason maintenance.

Finally, it is also possible to provide a result for infobase change which is reminiscent of the Harper Identity (the identity (Def $-$ from $*$)).

PROPOSITION 3.10. *Let $\circledast$ be a basic infobase revision, and let $\ominus$ be an infobase change operation such that $IB \ominus \alpha \approx \overleftarrow{IB \circledast \neg\alpha}$. Then $\ominus$ is a basic infobase contraction.*

PROOF. Follows from the fact that there is a basic infobase contraction *circleddash'* such that $IB \circledast \neg\alpha = (IB \ominus' \neg\neg\alpha) \bullet \neg\alpha$ and that $\alpha \equiv \neg\neg\alpha$. ■

To conclude this section we show that infobase change is able to accommodate Hansson's hamburger example in an appropriate fashion.

EXAMPLE 3.11. [10] "On a public holiday you are standing in the street in a town that has two hamburger restaurants. Let us consider the subset of your belief set that represents your beliefs about whether or not each of these two restaurant is open.

When you meet me, eating a hamburger, you draw the conclusion that at least one of the restaurant is open ($a \vee b$). Further, seeing from a distance that one of the two restaurants has its lights on, you believe that this particular restaurant is open (a). This situation can be represented by the set of beliefs $\{a, a \vee b\}$. When you have reached the restaurant however, you find a sign saying that it is closed all day. The lights are only turned on for the purposes of cleaning. You now have to include the negation of a, i.e. $\neg a$, into your belief set. The revision of $\{a, a \vee b\}$ to include $\neg a$ should still contain $a \vee b$, since you still have reason to believe that one of the two restaurants is open.

In contrast, suppose you had not met me or anyone else eating a hamburger. Then your only clue would have been the lights from the restaurant. The original belief system in this case can be represented by the set $\{a\}$. After finding out that the restaurant is closed, the resulting set should not contain $a \vee b$, since in this case you have no reason to believe that one of the restaurants is open." ■

Let L be the propositional language generated by the two atoms p and q, where $U = \{00, 01, 10, 11\}$. We let p denote the assertion that the restaurant whose lights are on is open, and we let q denote the assertion that the second restaurant is open. Now, let $IB = [p, p \vee q]$ and let $IC = [p]$. Since $IB^{-\neg\neg p} = \{p\}$, it follows from propositions 2.11 and 2.13 that for every basic infobase revision $\circledast$, there is a β in $IB \circledast \neg p$ such that $\beta \equiv p \vee q$. Furthermore, since $IC^{-\neg\neg p} = IC$, it follows that for every basic infobase revision $\circledast$, $IC \circledast \neg p \approx [\top, \neg p] \approx [\neg p]$. As our intuition suggests, revising IB by $\neg p$ yields an infobase containing $p \vee q$ (or something locially equivalent to it). In contract, a revision of IC by $\neg p$ does not contain such a wff. Nor, for that matter, does $p \vee q$ follow logically from the infobase resulting from a $\neg p$-revision of IC.

4. Iterated infobase change

Although an infobase IB induces the unique theory contraction $-_{IB}$, infobases do not contain enough information to determine a basic infobase contraction or revision. To do that, we also need a relevance selection function rs. Once rs is fixed, though, we are dealing with a specific basic infobase contraction and revision, which allows for the possibility of iterated infobase change. In this section we investigate whether iterated infobase change measures up to the postulates supplied by Darwiche et al. [4, 5] and Lehmann [14]. To do so, we have to work on the level of *epistemic states*.[9] Following Darwiche and Pearl we assume that every epistemic state Φ has associated with it an ordered pair consisting of a belief set $K(\Phi)$ and a $K(\Phi)$-faithful total preorder $\preceq_\Phi$. To bring infobase change into this framework, we assume that it is possible to extract a unique infobase IB_Φ from every epistemic state Φ. This implies that $K(\Phi) = \mathrm{Cn}(S(IB_\Phi))$ and that $\preceq_\Phi$ is identical to the IB_Φ-induced faithful total preorder $\preceq_{IB_\Phi}$. Note that the incorporation

[9] The use of epistemic states have been advocated by a number of authors, including Darwiche and Pearl [5] and Lehmann [14].

of infobases requires of epistemic states to have a richer structure than ordered pairs of the form $(K(\Phi), \preceq_\Phi)$. This is because infobases contain more information than such ordered pairs. For example, letting $IB = [p, q]$ and $IC = [p \wedge q, p \vee q]$, it is easy to check that $\mathrm{Cn}(S(IB)) = \mathrm{Cn}(S(IC))$, and that $\preceq_{IB}$ and $\preceq_{IC}$ are identical. Furthermore, since Darwiche and Pearl operate under the assumption of a finitely generated propositional language L, we shall do the same for the rest of this section, and from this it is easy to establish that every possible ordered pair of the kind described above can be obtained from some infobase.

LEMMA 4.1. *For every ordered pair of the form $(K, \preceq)$ where K is a belief set and $\preceq$ is a K-faithful total preorder, there is an infobase IB such $\preceq$ and $\preceq_{IB}$ are identical, and $K = \mathrm{Cn}(S(IB))$.*

PROOF. Pick any ordered pair of the form $(K, \preceq)$ where K is a belief set and $\preceq$ is a K-faithful total preorder. Since L is a finitely generated propositional language, U contains a finite number of interpretations. The total preorder $\preceq$ thus partitions U into a finite number of subsets (blocks). Let us assume that there are n such blocks. We assign each of them a unique index from 1 to n according to their relative positions in $\preceq$, leaving us with the n indexed blocks $P_1, \ldots, P_n$. That is, for $1 \leq i, j \leq n$, $i < j$ iff for every $u \in P_i$ and every $v \in P_j$, $u \prec v$. Now, for any $V \subseteq U$, let α_V be some some wff that axiomatises V. (Since L is finitely generated, such a wff always exists.) For $1 \leq i \leq n$, let $\beta_i \equiv \alpha_V$ where $V = \bigcup_{1 \leq j \leq i} P_j$. We define an infobase IB as follows: if $\perp \in K$, then IB contains exactly one instance of each of the wffs in $\{\perp\} \cup \bigcup_{1 \leq i \leq n} \{\beta_i\}$, otherwise IB contains exactly one instance of each of the wffs in $\bigcup_{1 \leq i \leq n} \{\beta_i\}$. It is easily verified that $\preceq$ and $\preceq_{IB}$ are identical, and that $\mathrm{Cn}(S(IB)) = K$. ■

More importantly, perhaps, is the fact that the extra information contained in infobases plays an important role in the process of infobase change, as the next example shows.

EXAMPLE 4.2. Let $\ominus$ be the basic infobase contraction obtained from the relevance selection function rs, where $rs(IB, \alpha) = IB^{-\alpha}$, for every $IB \in \mathcal{IB}$ and every $\alpha \in L$, and let $\circledast$ be the basic infobase revision defined in terms of $\ominus$ using (Def $\circledast$ from $\ominus$). Now, let $IB = [p, q]$ and let $IC = [p \wedge q, p, q, p \vee q, p \rightarrow q, q \rightarrow p]$. Clearly $\mathrm{Cn}(S(IB)) = \mathrm{Cn}(S(IC))$ and it is also easy to see that $\preceq_{IB}$ and $\preceq_{IC}$ are identical, and are represented graphically in figure 2. Yet, it can be verified that $IB \circledast (p \wedge \neg q) \approx [p, \top, p \wedge \neg q]$ and that $IC \circledast (p \wedge \neg q) \approx [p, p, p \vee q, p \vee q, \top, q \rightarrow p, p \wedge \neg q]$. So $IB \circledast (p \wedge \neg q)$ and $IC \circledast (p \wedge \neg q)$ induce different faithful total preorders, as can be seen in figure 3. ■

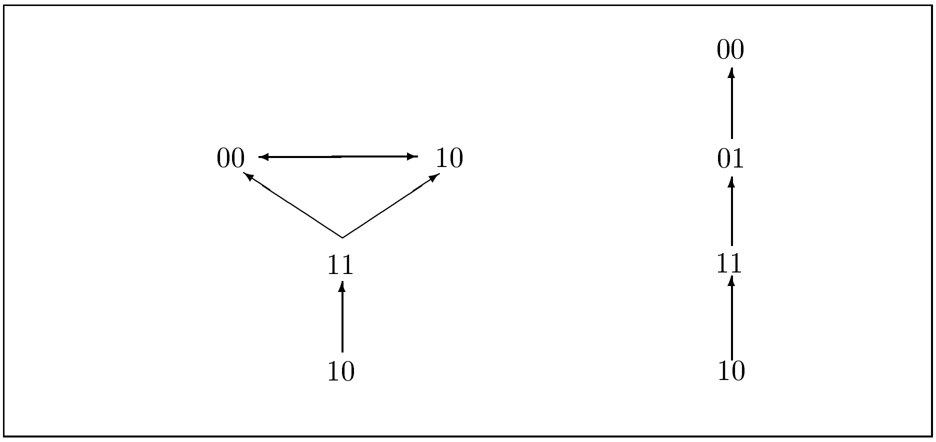

Figure 3. A graphical representation of the total preorders used in example 4.2. On the left is the $(IB \circledast (p \wedge \neg q))$-induced faithful total preorder and on the right the $(IC \circledast (p \wedge \neg q))$-induced faithful total preorder. As usual, the applicable preorder is the reflexive transitive closure of the relation determined by the arrows.

Having seen that infobases have a richer structure than ordered pairs of the form $(K(\Phi), \preceq_\Phi)$, we now turn to the definition of revision on epistemic states in terms of basic infobase revision.

(Def $\divideontimes$ from $\circledast$) $$\left[\begin{array}{rcl} K(\Phi \divideontimes \alpha) & = & \mathrm{Cn}(IB_\Phi \circledast \alpha) \\ \preceq_{\Phi \divideontimes \alpha} & = & \preceq_{(IB_\Phi \circledast \alpha)} \end{array}\right]$$

Definition 4.3. We refer to the revision on epistemic states defined in terms of a basic infobase revision $\circledast$ using (Def $\divideontimes$ from $\circledast$) as the $\circledast$-*associated* revision on epistemic states.

4.1. DP-revision

In two influential papers, Darwiche et al. [4, 5] argue that belief change ought to be conducted on the level of epistemic states. They concern themselves only with revision operations on epistemic states whose associated operations on belief sets are AGM revision operations, and propose the following four postulates for iterated revision:

(DP1) If $\alpha \vDash \beta$ then $K((\Phi \divideontimes \beta) \divideontimes \alpha) = K(\Phi \divideontimes \alpha)$

(DP2) If $\alpha \vDash \neg\beta$ then $K((\Phi \divideontimes \beta) \divideontimes \alpha) = K(\Phi \divideontimes \alpha)$

(DP3) If $\beta \in K(\Phi \divideontimes \alpha)$ then $\beta \in K((\Phi \divideontimes \beta) \divideontimes \alpha)$

(DP4) If $\neg\beta \notin K(\Phi \divideontimes \alpha)$ then $\neg\beta \notin K((\Phi \divideontimes \beta) \divideontimes \alpha)$

They then proceed to show that each one of the postulates (DP$\circledast$1) to (DP$\circledast$4) can be represented semantically as follows:

(DPR$\circledast$1) If $u \Vdash \alpha$ and $v \Vdash \alpha$ then $u \preceq_\Phi v$ iff $u \preceq_{\Phi \circledast \alpha} v$

(DPR$\circledast$2) If $u \Vdash \neg\alpha$ and $v \Vdash \neg\alpha$ then $u \preceq_\Phi v$ iff $u \preceq_{\Phi \circledast \alpha} v$

(DPR$\circledast$3) If $u \Vdash \alpha$ and $v \Vdash \neg\alpha$ then $u \prec_\Phi v$ only if $u \prec_{\Phi \circledast \alpha} v$

(DPR$\circledast$4) If $u \Vdash \alpha$ and $v \Vdash \neg\alpha$ then $u \preceq_\Phi v$ only if $u \preceq_{\Phi \circledast \alpha} v$

THEOREM 4.4. [5] *Let $\circledast$ be a revision operation on epistemic states whose associated operation on belief sets is an* AGM *revision operation. Then $\circledast$*

$$\textit{satisfies} \left\{ \begin{array}{c} (\mathrm{DP} \circledast 1) \\ (\mathrm{DP} \circledast 2) \\ (\mathrm{DP} \circledast 3) \\ (\mathrm{DP} \circledast 4) \end{array} \right\} \quad \textit{iff} \quad \textit{it satisfies} \left\{ \begin{array}{c} (\mathrm{DPR} \circledast 1) \\ (\mathrm{DPR} \circledast 2) \\ (\mathrm{DPR} \circledast 3) \\ (\mathrm{DPR} \circledast 4) \end{array} \right\}.$$

When placed in this framework, basic infobase revision yields favourable results. The revisions on epistemic states associated with basic infobase revisions satisfy all but the first one of the four DP-postulates. The satisfaction of these three DP-postulates rely on the following two simple results.

LEMMA 4.5. *Let $\circledast$ be a basic infobase revision and let rs be the relevance selection function from which $\circledast$ is obtained.*

1. *If $v \in M(\neg\alpha)$ then, for every β in IB, $v \in M(\beta)$ iff $v \in M(w^{rs(IB,\neg\alpha)}_{(IB,\neg\alpha)}(\beta))$.*
2. *For every β in IB, if $v \in M(\beta)$ then $v \in M(w^{rs(IB,\neg\alpha)}_{(IB,\neg\alpha)}(\beta))$.*

PROOF. By proposition 2.13, $M\left(w^{rs(IB,\neg\alpha)}_{(IB,\neg\alpha)}(\beta)\right) = M(\beta) \cup N^{rs(IB,\neg\alpha)}_\beta(\neg\neg\alpha)$ for every β in *IB*.
1. Follows from the fact that $N^{rs(IB,\neg\alpha)}_\beta(\neg\neg\alpha) \subseteq M(\alpha)$ for every β in *IB*.
2. Follows from the fact that $M(\beta) \subseteq M\left(w^{rs(IB,\neg\alpha)}_{(IB,\neg\alpha)}(\beta)\right)$. ■

PROPOSITION 4.6. *Let $\circledast$ be a basic infobase revision, and let $\circledast$ be the $\circledast$-associated revision on epistemic states. Then $\circledast$ satisfies* (DP2)–(DP4), *but does not necessarily satisfy* (DP1).

PROOF. To show that $\circledast$ does not necessarily satisfy (DP1), let L be generated by the atoms p and q, where $U = \{00, 01, 10, 11\}$. Let $IB_\Phi = [p \leftrightarrow q, p \vee \neg q, \neg p \vee \neg q, \neg q]$ and let $\circledast$ be the basic infobase revision obtained from the relevance selection function rs for which $rs(IB, \alpha) = IB^{-\alpha}$ for every $IB \in \mathcal{IB}$ and every $\alpha \in L$. It can be verified that

$$\begin{aligned} IB_\Phi \circledast (p \vee q) &\approx [p \vee \neg q, \neg p \vee \neg q, \neg q, p \vee q], \\ K((\Phi \divideontimes (p \vee q)) \divideontimes q) &= \mathrm{Cn}(S((IB_\Phi \circledast (p \vee q)) \circledast q)) = \mathrm{Cn}(q), \text{ and} \\ K(\Phi \divideontimes q) &= \mathrm{Cn}(S(IB_\Phi \circledast q)) = \mathrm{Cn}(p \wedge q). \end{aligned}$$

So $q \vDash p \vee q$, but $K((\Phi \divideontimes (p \vee q)) \divideontimes q) \neq K(\Phi \divideontimes q)$, which is a violation of (DP1).

For (DP2)–(DP4), it suffices, by theorem 4.4, to show that $\divideontimes$ satisfies (DPR2)–(DPR4). Let rs be the relevance selection function from which $\circledast$ is obtained and pick any epistemic state Φ.

For (DPR2), observe that since $IB_\Phi \circledast \alpha$ is obtained by replacing every wff β in IB_Φ with $w^{rs(IB_\Phi,\neg\alpha)}_{(IB_\Phi,\neg\alpha)}(\beta)$ and then adding α, it follows from part (1) of lemma 4.5 that $u_{IB_\phi} = u_{IB_\Phi \circledast \alpha}$ (where u_{IB_ϕ} and $u_{IB_\Phi \circledast \alpha}$ are the IB_Φ-number and the $(IB_\Phi \circledast \alpha)$-number of u), for every $u \in M(\neg\alpha)$. And since $\preceq_\Phi$ is the IB_Φ-induced faithful total preorder, and $\preceq_{\Phi \divideontimes \alpha}$ is the $(IB_\Phi \circledast \alpha)$-induced faithful total preorder, it then follows that $u \preceq_\Phi v$ iff $u \preceq_{\Phi \divideontimes \alpha} v$ for every $u, v \in M(\neg\alpha)$. So (DPR2) is satisfied.

For (DPR3) and (DPR4), note that part (2) of lemma 4.5 ensures that $u_{IB_\phi} \leq u_{IB_\Phi \circledast \alpha}$. Combined with part (1) of lemma 4.5, it then follows for every $u \in M(\alpha)$ and every $v \in M(\neg\alpha)$, that if $u_{IB_\phi} > v_{IB_\phi}$ then $u_{IB_\Phi \circledast \alpha} > v_{IB_\Phi \circledast \alpha}$. So, for every $u \in M(\alpha)$ and every $v \in M(\neg\alpha)$, if $u \prec_\Phi v$ then $u \prec_{\Phi \divideontimes \alpha} v$, which means that (DPR3) holds. Similarly, from parts (1) and (2) of lemma 4.5 it follows for every $u \in M(\alpha)$ and every $v \in M(\neg\alpha)$, that if $u_{IB_\phi} \geq v_{IB_\phi}$ then $u_{IB_\Phi \circledast \alpha} \geq v_{IB_\Phi \circledast \alpha}$. So, for every $u \in M(\alpha)$ and every $v \in M(\neg\alpha)$, if $u \preceq_\Phi v$ then $u \preceq_{\Phi \divideontimes \alpha} v$; that is, (DPR4) holds. ■

Observe that this is in marked contrast with the version of infobase revision described in [15] which does not satisfy any of these postulates.

It is our contention that the violation of (DP1) by basic infobase revision is an indication that this postulate is perhaps too restrictive to accommodate a wide range of rational forms of revision. Below we give a realistic example in support of this claim.[10]

EXAMPLE 4.7. I have a circuit containing two components; an adder and a multiplier. I have made three independent observations about these components: (1) The adder is working, (2) the multiplier is working, and (3) if the adder doesn't work then the multiplier also doesn't work. Another observation now indicates that at least one of the two components is not working. In trying to incorporate this new information, we have to discard (or weaken) at least one of the first two observations. Moreover, we cannot

[10] The example was inspired by a similar example of Darwiche et al. [5].

retain both observations (2) and (3), for they imply observation (1). So it seems reasonable to retain the belief that the adder is working and the belief that a broken adder implies a broken multiplier. Together with the new information that at least one of the components is broken, it then follows that it is the multiplier that is broken.

This line of reasoning can be formalised by using the two atoms a (indicating that the adder is working) and m (indicating that the multiplier is working). My initial infobase then looks like this: $IB = [a, m, \neg a \to \neg m]$. Figure 4 contains a graphical representation of the IB-induced faithful total preorder $\preceq_{IB}$. It is easily verified that for any basic infobase revision $\circledast$, $\mathrm{Cn}(S(IB \circledast \neg(a \wedge m))) = \mathrm{Cn}(a \wedge \neg m)$, which means that m should be discarded and that a and $\neg a \to \neg m$ should be retained. But what should the weakened version of the discarded wff m look like? One reasonable option is to discard it completely, or, what amounts to the same thing, to weaken it so that it becomes logically valid. Formally, this can be accomplished as follows. Let rs be a relevance selection function such that $rs(IB, a \wedge m) = IB^{-(a \wedge m)} = \{m\}$. Since $IB^{-(a \wedge m)}$ is $(IB, \alpha \wedge m)$-relevant, there is such an rs. Now consider the basic infobase contraction $\ominus$ which is obtained using rs. It can be verified that $IB \ominus \neg\neg(a \wedge m) \approx IB \ominus (a \wedge m) \approx [a, \top, \neg a \to \neg m]$ and therefore $IB \circledast \neg(a \wedge m) \approx [a, \top, \neg a \to \neg m, \neg(a \wedge m)]$, where $\circledast$ is the basic infobase revision defined in terms of $\ominus$ using (Def $\circledast$ from $\ominus$). Figure 4 contains a graphical representation of the $IB \circledast \neg(a \wedge m)$-induced faithful total preorder $\preceq_{IB}$. To see that the revision $\divideontimes$ defined in terms of $\circledast$ using (Def $\divideontimes$ from $\circledast$) violates (DP1), observe that $\mathrm{Cn}(S(IB \circledast \neg a)) = \mathrm{Cn}(\neg a)$, but that $\mathrm{Cn}(S((IB \circledast \neg(a \wedge m)) \circledast \neg a)) = \mathrm{Cn}(\neg a \wedge \neg m)$. So $K((\Phi \divideontimes \neg(a \wedge m)) \divideontimes \neg a) \neq K(\Phi \divideontimes \neg a)$ even though $\neg a \vDash \neg(a \wedge m)$ where Φ is an epistemic state such that $IB_\Phi = IB$. ■

There is a form of basic infobase revision which always satisfies (DP1). It corresponds to the coherentist approach to infobase change.

Definition 4.8. A *coherentist* basic infobase revision $\circledast$ is a basic infobase revision such that $rs(IB, \alpha) = IB$ for every $\alpha \in L$, for the relevance selection function rs from which $\circledast$ is obtained.

To show that a coherentist basic infobase revision satisfies (DP1) we need the following two lemmas.

Lemma 4.9. *For every $u \in Min_{\preceq_{IB}}(\alpha)$, $N_u^{IB}(\alpha) \subseteq Min_{\preceq_{IB}}(\alpha)$.*

Proof. Pick any $u \in Min_{\preceq_{IB}}(\alpha)$ and any $v \in N_u^{IB}(\alpha)$. By definition, $v \in M(\alpha)$, and u and v satisfy exactly the same wffs in IB. So the IB-numbers of u and v are the same, and therefore $v \in Min_{\preceq_{IB}}(\alpha)$. ■

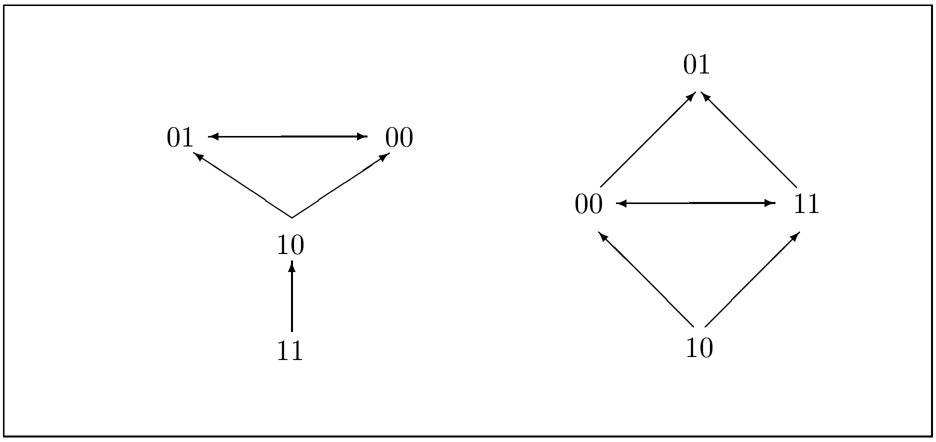

Figure 4. A graphical representation of the total preorders used in example 4.7.On the left is the IB-induced faithful total preorder and on the right the $(IB \circledast \neg(a \wedge m))$-induced faithful total preorder. As usual, the applicable preorder is the reflexive transitive closure of the relation determined by the arrows.

LEMMA 4.10. *If* $v \in M(\alpha) \setminus Min_{\preceq_{IB}}(\alpha)$ *then, for all* β *in* IB, $v \in M(\beta)$ *iff* $v \in M(w^{IB}_{(IB,\neg\alpha)}(\beta))$.

PROOF. Pick any $v \in M(\alpha) \setminus Min_{\preceq_{IB}}(\alpha)$ and any β in IB. It follows by proposition 2.13 that $M(\beta) \subseteq M(w^{IB}_{(IB,\neg\alpha)}(\beta))$ and so $v \in M(\beta)$ implies $v \in M(w^{IB}_{(IB,\neg\alpha)}(\beta))$. Conversely, suppose that $v \in M(w^{IB}_{(IB,\neg\alpha)}(\beta))$. By lemma 4.9, $v \notin N^{IB}_{\beta}(\alpha)$, and it therefore follows from proposition 2.13 that $v \in M(\beta)$. ■

PROPOSITION 4.11. *Let* $\circledast$ *be the coherentist basic infobase revision and let* $\divideontimes$ *be the revision on epistemic states defined in terms of* $\circledast$ *using* (Def $\divideontimes$ from $\circledast$). *Then* $\divideontimes$ *satisfies* (DP1).

PROOF. By theorem 4.4, it suffices to show that $\divideontimes$ satisfies (DPR1). Let Φ be any epistemic state. So $\preceq_\Phi$ is the IB_Φ-induced faithful total preorder. We have to show that $u \preceq_\Phi v$ iff $u \preceq_{\Phi \divideontimes \alpha} v$ for every $u, v \in M(\alpha)$.

Recall from definitions 2.12 and 3.6 that $IB_\Phi \circledast \alpha$ is obtained by replacing every wff β in IB_Φ with $w^{IB_\Phi}_{(IB_\Phi,\neg\alpha)}(\beta)$ and then adding α. From lemma 4.10 it follows that the $(IB_\Phi \circledast \alpha)$-number of u is one more than the IB_Φ-number of u, for every $u \in M(\alpha) \setminus Min_{\preceq_\Phi}(\alpha)$. So $u \preceq_\Phi v$ iff $u \preceq_{\Phi \divideontimes \alpha} v$ for every $u, v \in M(\alpha) \setminus Min_{\preceq_\Phi}(\alpha)$.

Next, observe that the IB_Φ-number of every $u \in Min_{\preceq_\Phi}(\alpha)$ is greater than the IB_Φ-number of every $v \in M(\alpha) \setminus Min_{\preceq_\Phi}(\alpha)$. Moreover, by part

(2) of corollary 3.9 it follows that $M(S(IB_\Phi \circledast \alpha)) = Min_{\preceq_\Phi}(\alpha)$. So the $(IB_\Phi \circledast \alpha)$-number of every $u \in Min_{\preceq_\Phi}(\alpha)$ is greater than the $(IB_\Phi \circledast \alpha)$-number of every $v \in M(\alpha) \setminus Min_{\preceq_\Phi}(\alpha)$. Therefore $u \preceq_\Phi v$ iff $u \preceq_{\Phi \circledast \alpha} v$ for every $u \in Min_{\preceq_\Phi}(\alpha)$ and every $v \in M(\alpha) \setminus Min_{\preceq_\Phi}(\alpha)$, and $u \preceq_\Phi v$ iff $u \preceq_{\Phi \circledast \alpha} v$ for every $v \in Min_{\preceq_\Phi}(\alpha)$ and every $u \in M(\alpha) \setminus Min_{\preceq_\Phi}(\alpha)$.

Finally, observe that elements of $Min_{\preceq_\Phi}(\alpha)$ all have the same IB_Φ-number, and since $M(S(IB_\Phi \circledast \alpha)) = Min_{\preceq}(\alpha)$, the elements of $Min_{\preceq_\Phi}(\alpha)$ all have the same $(IB_\Phi \circledast \alpha)$-number as well. So $u \preceq_\Phi v$ iff $u \preceq_{\Phi \circledast \alpha} v$ for every $u, v \in Min_{\preceq_\Phi}(\alpha)$, which means we are done. ■

4.2. L-revision

Lehmann [14] considers iterated belief revision in the context of finite sequences of revisions. He extends the notion of a revision $\circledast$ on epistemic states to a revision by a finite sequence of wffs. $\Phi \circledast \sigma$ then refers to the iterated revision of Φ by the wffs in σ, and if σ is the empty sequence, $\Phi \circledast \sigma$ is just the epistemic state Φ. A wff α is identified with a sequence of length one. Considering only sequences of *satisfiable* wffs, Lehmann proposes the following postulates for iterated revision.

(L$\circledast$1) $K(\Phi) = \mathrm{Cn}(K(\Phi))$

(L$\circledast$2) $\alpha \in K(\Phi \circledast \alpha)$

(L$\circledast$3) $K(\Phi \circledast \alpha) \subseteq K(\Phi) + \alpha$

(L$\circledast$4) If $\alpha \in K(\Phi)$ then $K(\Phi \circledast \sigma) = K(\Phi \circledast (\alpha \bullet \sigma))$

(L$\circledast$5) If $\alpha \vDash \beta$ then $K(\Phi \circledast (\beta \bullet \alpha \bullet \sigma)) = K(\Phi \circledast (\alpha \bullet \sigma))$

(L$\circledast$6) $K(\Phi) \neq \mathrm{Cn}(\bot)$

(L$\circledast$7) $K(\Phi \circledast (\neg\alpha \bullet \alpha)) \subseteq K(\Phi) + \alpha$

(L$\circledast$8) If $\neg\beta \notin K(\Phi \circledast \alpha)$ then
$K(\Phi \circledast (\alpha \bullet \beta \bullet \sigma)) = K(\Phi \circledast (\alpha \bullet \alpha \wedge \beta \bullet \sigma))$

It is easily verified that the revision $\circledast$ on epistemic states obtained in terms of a basic infobase revision using (Def $\circledast$ from $\circledast$) satisfies (L$\circledast$1), (L$\circledast$2), (L$\circledast$3) and (L$\circledast$6). It can also be verified that (L$\circledast$7) is a weakened version of (DP2) and it thus follows from proposition 4.6 that $\circledast$ also satisfies (L $\circledast$7). It does not necessarily satisfy (L$\circledast$4), (L$\circledast$5) and (L$\circledast$8), though, as the following example shows.

EXAMPLE 4.12. Let $\circledast$ be the basic infobase revision obtained from the relevance selection function rs for which $rs(IB, \alpha) = IB^{-\alpha}$ for every $IB \in \mathcal{IB}$ and every $\alpha \in L$.

1. Let $IB = [p \wedge \neg q, p \vee q]$. Clearly $IB \circledast p \approx [p \wedge \neg q, p \vee q, p]$. It can be verified that $\mathrm{Cn}(S((IB \circledast p) \circledast q)) = \mathrm{Cn}(p \wedge q)$, but that $\mathrm{Cn}(S(IB \circledast q)) = \mathrm{Cn}(q)$. Taking p as α and q as the sequence of wffs σ, this is a violation of (L⊛4).
2. Let $IB = [p \leftrightarrow q, p \vee \neg q, \neg p \vee \neg q, \neg q]$. It can be verified that $IB \circledast q \approx [p \leftrightarrow q, p \vee \neg q, p \vee \neg q, q]$, $IB \circledast p \vee q \approx [p \vee \neg q, \neg p \vee \neg q, \neg q, p \vee q]$, $(IB \circledast p \vee q) \circledast q \approx [p \vee q, q]$, $\mathrm{Cn}(S(((IB \circledast p \vee q) \circledast q) \circledast \neg q)) = \mathrm{Cn}(p \wedge \neg q)$, and $\mathrm{Cn}(S((IB \circledast q) \circledast \neg q)) = \mathrm{Cn}(\neg p \wedge \neg q)$. Taking $p \vee q$ as β, q as α, and $\neg q$ as the sequence of wffs σ, this constitutes a violation of (L⊛5).
3. Let $IB = [p \vee q, p \vee \neg q]$. Clearly $IB \circledast p \approx [p \vee q, p \vee \neg q, p]$, $(IB \circledast p) \circledast q = [p \vee q, p \vee \neg q, p, q]$, and $(IB \circledast p) \circledast p \wedge q = [p \vee q, p \vee \neg q, p, p \wedge q]$. It can be verified that $\mathrm{Cn}(S(((IB \circledast p) \circledast q) \circledast \neg p)) = \mathrm{Cn}(\neg p \wedge q)$, and $\mathrm{Cn}(S(((IB \circledast p) \circledast p \wedge q) \circledast \neg p)) = \mathrm{Cn}(\neg p)$. With p as α, q as β, and $\neg p$ as the sequence of wffs σ, it follows that (L⊛8) is violated. ■

An analysis of this example suggests that, unlike the DP-postulates, (L⊛4), (L⊛5) and (L⊛8) are fundamentally incompatible with basic infobase revision.

5. Conclusion

In this paper we have extended the initial infobase proposal of Meyer et al. [15] in two ways. Firstly, we have replaced the initial proposal for a definition of infobases as finite sets of wff, with one in which an infobase is defined as a finite sequence of wffs. The fact that iterated infobase change, as defined in this paper, satisfies the DP-postulates, unlike the initial version in [15], serves as confirmation that the current proposal is a step in the right direction. Secondly, we have generalised the initial infobase proposal by presenting a whole spectrum of infobase change operations obtained from a given infobase. Notwithstanding these advances, much still needs to be done. Two obvious extensions that still need to be developed has already been hinted at by Meyer et al. [15]. Both involve the introduction of orderings of epistemic relevance in the spirit of Nebel [18, 19, 20]. It also remains to be seen how basic infobase change fits into a more general theory of base change.

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